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A form-factor approach to finite-temperature correlation functions in $c = 1$ CFT

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Abstract
The excitation spectrum of specific conformal field theories (CFT) with central charge $c = 1$ can be described in terms of quasi-particles with charges $Q = -p, +1$ and fractional statistics properties. Using the language of Jack polynomials, we compute form factors of the charge density operator in these CFTs. We study a form-factor expansion for the finite-temperature density–density correlation function, and find that it shows a quick convergence to the exact result. The low-temperature behaviour is recovered from a form factor with $p + 1$ particles, while the high-temperature limit is recovered from states containing no more than three particles.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

One of the extraordinary features of low-dimensional condensed matter systems is that they possess fractional characteristics. Very soon after the discovery of the fractional quantum Hall effect, it was understood that the excitations of such states of matter carry both fractional charge and fractional exchange statistics. Later, Haldane [1] proposed an interpretation of these statistics as being exclusion statistics, a concept that can be defined for any dimension. It states that the presence of a particle i restricts the dimension of the Hilbert space that is available to another particle j by an amount g_{ij}, called the exclusion statistics parameter. This generalizes the Pauli principle for fermions.

The low-energy effective theory for a fractional quantum Hall (fqH) system is a chiral conformal field theory (CFT), and it has been found [2] that these edge theories can be analysed in terms of quasi-particles with fractional charge and statistics. More general chiral

RCFT spectra can be built out of fractional statistics excitations through what is known as the 'universal chiral partition function' [3]. In the following, we will be interested in the edge CFT of a principal Laughlin state at filling fraction $\nu = 1/p$, where p is an integer. The edge theory is that of a compactified boson at radius $R^2 = p$. The chiral Hilbert space of the theory has been understood [2] as a collection of multi-particle states built out of fundamental quasi-particle excitations. For $\nu = 1/p$, they are edge electrons, of charge $(-e)$, and edge quasi-holes, of charge (e/p), both described by primary fields of the CFT.

It was shown [2, 4] that these quasi-particle excitations obey fractional (exclusion) statistics based on the matrix

$$\boldsymbol{g} = \begin{pmatrix} p & 0 \\ 0 & 1/p \end{pmatrix}. \tag{1}$$

It means that edge electrons and edge quasi-holes are p- and $1/p$-ons, with no mutual exclusion. The $g = p$ and $g = 1/p$ particles enjoy duality properties, in that their thermodynamic distribution functions are dual to each other.

These properties are reminiscent of the Calogero–Sutherland (CS) model. Indeed, the chiral CFT for $\nu = 1/p$ can be identified with the continuum limit of a CS model with interaction parameter equal to p [5]. The relation between the fqH basis and the CS basis, defined in terms of Jack symmetric polynomials, has been worked out in [6]. The Jack polynomial technology is then available to compute the action of observables on multi-particle states. From there, form factors can be obtained, as has been shown in [6]. Proceeding in this manner, one does not have to rely on form-factor axioms for integrable field theories (IFT), since explicit computations in a regularized CFT can be performed.

It has been proposed that, in integrable field theories, correlation functions at finite temperature can be represented in a form-factor expansion. In such an expansion one adds the contributions to the correlator coming from specific multi-particle states, each weighted by an appropriate multi-particle distribution function. There is an ongoing discussion on how precisely these ideas can be implemented for IFTs [7–12]. In the context of $c = 1$ CFTs, a form-factor expansion for a correlator can be compared with the exact result obtained thanks to the KZ equations. In [6], the expansion of a specific electron Green function in the $\nu = 1/p$ chiral CFT was shown to converge quickly in terms of the number of excitations considered. We report here on progress made in the same direction for the density–density correlation function, paying special attention to low- and high-temperature limits. At the technical level our new results include

(i) an interpretation of the quasi-particle states with non-monotonic ordering of the m_i and n_j, in terms of 'vanishing' and 'scattered states' (see equation (26)),
(ii) exact results for the action of the density operator on states with up to three particles,
(iii) a precise definition of the irreducible part of form factors with more than a single quasi-hole, in which case the sector index 'Q' plays an important role (see equation (41)) and
(iv) consistency checks using the sum-rule equation (17), based on the Sugawara form of the Virasoro operator in a CFT with $U(1)$ symmetry.

Analysing the form-factor expansion for the density–density correlator, we demonstrate that the leading asymptotic behaviour for low temperature is given by the 'exciton' configuration with a single electron and p quasi-holes. More surprisingly, we show by explicit computation that the high-temperature limit (meaning the $\beta\varepsilon \to 0$ limit) of the Green function $\langle\rho(-\varepsilon)\rho(\varepsilon)\rangle_T$, is recovered from form factors containing no more than three particles. This remarkable result is established with the help of the identity equation (54) satisfied by the thermodynamic distribution function $\bar{n}_g(\varepsilon)$ for particles satisfying exclusion statistics with parameter g.

Our computations in this paper can be viewed as a dry run for the computation of non-trivial transport computations in systems that are not CFTs but that do have quasi-particles with well-defined exclusion statistics properties (examples are finite-N CS models or Haldane–Shastry spin chains). For the formalism to become of practical use, it needs further streamlining, possibly by making and exploiting a connection with the axiomatic approach to form factors, based on scattering data in IFT.

This paper is organized as follows. In section 2, we introduce our notations by recalling the fqH basis construction in [6]. We give the basic properties of the excitations and their expressions in terms of Jack polynomials. In section 3, we give the expressions for form factors up to three particles for the density operator. In section 4 we turn to the form-factor expansion for the density–density correlation function. Using the results of the preceding section, we show what are the different contributions to the low- and high-temperature limits. Exact numerical results will sustain our arguments in favour of a quick convergence for the full correlation function.

2. Fractional quantum Hall systems and Jack polynomials

2.1. fqH basis

The effective low-energy theory for a $\nu = 1/p$ fqH system is a compactified boson with radius $R^2 = p$, which is a chiral $c = 1$ CFT. The Hilbert space for this theory is obtained as a collection of sectors of zero modes. The sectors are labelled by an integer Q, which is the $U(1)$ electric charge measured in units of e/p. The partition function is

$$Z = \mathrm{Tr}(q^{L_0}) = \sum_{Q=-\infty}^{\infty} \frac{q^{Q^2/2p}}{(q)_\infty} \tag{2}$$

where $(q)_N = \prod_{l=1}^{N}(1 - q^l)$.

This Hilbert space can be understood as a collection of multi-particle states, the fundamental quasi-particles being the edge electron and the edge quasi-hole, of charge $Q = -p$ and $Q = 1$, respectively. They are described by the conformal primary fields

$$J^{(-p)}(z) = \sum_t J_{-t} z^{t-p/2} \qquad \phi^{+}(z) = \sum_s \phi_{-s} z^{s-1/2p}. \tag{3}$$

The Fourier modes of these operators can be interpreted as creation operators. The independent multi-particle states that generate the chiral Hilbert space were identified to be

$$|m_M, \ldots m_1; n_N \ldots n_1\rangle^Q \equiv J_{-(2M-1)p/2+Q-m_M} \ldots J_{-p/2+Q-m_1} \phi_{-(2N-1)/2p-Q/p-n_N} \ldots \phi_{-1/2p-Q/p-n_1} |Q\rangle \tag{4}$$

$$m_M \geqslant \cdots \geqslant m_1 \geqslant 0 \qquad n_N \geqslant \cdots \geqslant n_1 \geqslant 0 \quad (n_1 > 0 \text{ if } Q < 0) \tag{5}$$

where $|Q\rangle$ $(Q = -(p-1), \ldots, -1, 0)$ is the lowest-energy state of charge Q. The identification was proved through the equality of the partition functions.

2.2. Fractional statistics and duality

Fractional exclusion statistics is a tool introduced by Haldane [1] for the analysis of strongly correlated many-body systems. It is only based on the assumption that the Hilbert space is finite dimensional and extensive, i.e., particles are excitations of the considered condensed matter system, so it is a very generic concept. The statistics are encoded in a matrix $\boldsymbol{g} = (g_{ij})$

corresponding to the reduction of the available Hilbert space for particle of type i by filling a one-particle state by a particle of type j. This is then a generalization of the Pauli principle.

The thermodynamics for a gas of such particles have been worked out by Isakov, Ouvry and Wu (IOW) [13–15]. The one-particle grand canonical partition functions λ_i and distribution functions $\bar{n}_i$ are, respectively, given by the IOW equations

$$\left(\frac{\lambda_i - 1}{\lambda_i}\right)\prod_j \lambda_j^{g_{ij}} = \mathrm{e}^{\beta(\mu_i-\varepsilon)} \equiv z_i \tag{6}$$

$$\bar{n}_i(\varepsilon) = z_i \frac{\partial}{\partial z_i} \log \prod_j \lambda_j. \tag{7}$$

In the case of a fqH at $\nu = 1/p$ the statistical matrix is given by (1), where electrons and quasi-holes are non-exclusive to each other. For $p = 1$, we recover the Fermi–Dirac distribution functions, and for $p = 2$,

$$\bar{n}_2(\varepsilon) = \frac{1}{2}\left(1 - \frac{1}{\sqrt{1 + 4\,\mathrm{e}^{-\beta(\varepsilon-\mu)}}}\right) \qquad \bar{n}_{1/2}(\varepsilon) = \frac{2}{\sqrt{1 + 4\,\mathrm{e}^{2\beta(\varepsilon-\mu)}}}. \tag{8}$$

In general for g-ons, the distribution has a limiting value for $\varepsilon \to -\infty$ equal to $\bar{n}_g^{\mathrm{max}} = 1/g$ and an asymptotic behaviour for $\varepsilon \to \infty$ equal to $\mathrm{e}^{-\beta\varepsilon}$.

The transport properties of g-ons have been worked out in [16]. In the fqH case, one furthermore has a duality between g- and $1/g$-ons, appearing in the identity

$$g\bar{n}_g(\varepsilon) = 1 - \bar{n}_{1/g}(-\varepsilon/g)/g \tag{9}$$

related to the fact that the removal of a g-on corresponds to the creation of g $1/g$-ons. This agrees with the physics of edge-to-edge tunnelling in the fqH, in which the duality couples weak and strong backscattering. It also means the quasi-particle basis proposed in (4) is not unique. Iso [5] proposed a basis made out of particles of one kind only, filling up one-particle states with energies extending over both positive and negative values. One shall see that our 'excitation' picture is more practical to compute physical quantities.

The duality (9) can be used in the evaluation of thermodynamic quantities. In each Q sector, the partition function decomposes into a product of the partition function for electrons and that for quasi-holes. Then, the specific heat (or central charge) can be written as a sum over the electron and the quasi-hole contribution, and using (9) one finds $c = 1$ independent of p. Depending on the sign of an imposed voltage, the Hall conductance is given by an electron or by a quasi-hole expression, of the general form

$$G = \bar{n}_g^{\mathrm{max}} \frac{q^2}{h} = \frac{1}{p}\frac{e^2}{h} \tag{10}$$

where $q = e/p$ or $q = -e$ is the charge of the quasi-particle that carries the current. We shall see that the duality is also instrumental in the computation of form factors and the evaluation of the form-factor expansion.

2.3. Correspondence with Jack polynomials

As for the fqH multi-particle basis, it has been shown to be in one-to-one correspondence with the orthogonal eigenbasis of the Calogero–Sutherland (CS) model

$$H_{CS} = -\sum_{i=1}^{N} \frac{\partial^2}{\partial x_i^2} + \left(\frac{2\pi}{L}\right)^2 \sum_{i<j} \frac{2\lambda(\lambda-1)}{\sin^2(\pi x_{ij}/L)} \tag{11}$$

in the thermodynamic limit $N \to \infty$ and with interaction parameter $\lambda = p$. This Hamiltonian is best specified using a scalar field $\varphi(z)$ with $\partial\varphi(z) = \sum_n a_n z^{-n-1}$. In terms of this field, the CS Hamiltonian takes the form

$$H_{CS} = \frac{p-1}{p}\sum_{l=0}^{\infty}(l+1)(\mathrm{i}\sqrt{p}a_{-l-1})(\mathrm{i}\sqrt{p}a_{l+1}) + \frac{1}{3p}[(\mathrm{i}\sqrt{p}\partial\varphi)^3]_0. \tag{12}$$

It is then possible to build all the eigenstates of the Hamiltonian using multi-J and multi-ϕ quanta. It is found that the states (4) are not H_{CS} eigenstates, but that they rather act as head states that need to be supplemented by a tail of subleading states. The eigenbasis will then be denoted by

$$|\{m_i; n_j\}\rangle^Q = |m_i; n_j\rangle^Q + \cdots. \tag{13}$$

We refer to [6] for further details.

From a different perspective, the analysis in [5] has led to a basis of eigenstates of H_{CS}, specified as

$$|\{\mu\}, q\rangle = J^{1/p}_{\{\mu'\}}(\{p_n = \sqrt{p}a_{-n}\})|q\rangle \equiv J^{1/p}_{\{\mu'\}}|q\rangle \tag{14}$$

where the $U(1)$ charge q runs over all integers, and $\{\mu\}$ runs over all Young tableaus. $J^{1/p}_{\{\mu'\}}$ is called a Jack symmetric polynomial. They form a basis of the ring of symmetric functions with a given scalar product. Useful details about them are reported in the appendix.

A one-to-one correspondence between fqH states (13) and CS states (14) is obtained through the identification [6]

$$|\{m_i; n_j\}\rangle^Q = |(\{m\} + N^M) \cup \{n'\}, Q + N - pM\rangle \tag{15}$$

with $\{m\}$ the Young tableau built with the m_i quanta and $\{n'\}$ the dual Young tableau built with the n_j quanta.

Our strategy will be the following. We will set up the form-factor expansion in terms of the fqH basis, but do the actual computation of the form factors using the representation in terms of Jack polynomials, allowing us to exploit what is known about them. In the appendix, we present some 'Jack polynomial technology' that we have used for computing the various form factors. In the next section we give an overview of the form factors that we obtained.

3. Overview of form factors

In [6], form factors for the electron creation operator have been studied. Here we focus on the computation of form factors for the charge density operator in the fqH edge at $\nu = 1/p$. Its Fourier modes are $\mathrm{i}\rho_m = \mathrm{i}\sqrt{p}a_m = \mathrm{i}p_{-m}$, so that it has a nice interpretation in the language of Jack polynomials as a power sum. The form factors we are searching for are

$$\langle\{\nu\}, q|p_{-m}|\{\mu\}, q\rangle. \tag{16}$$

We shall obtain them by developing the product of a power sum and a Jack polynomial on the basis of Jack polynomials. In the previous works on the Calogero–Sutherland model [17], where only zero-temperature properties were computed, one only considered form factors with either the in or the out state equal to the vacuum. For such cases, only the expansion of power sums on a basis of Jack polynomials is needed. In the case of form factors of the general form (16), which appear in the form-factor expansion for finite-temperature correlators, it is necessary to use a less traditional approach, combining the expansion of the power sums in elementary symmetric functions and the Pieri formula written in the appendix. This allows in principle to compute *any* form factor. We have obtained closed, analytic expressions for a number of form factors with up to three particles. To obtain them, we have proceeded by evaluating a few simple examples (the smaller m's), conjecturing a general form, and then checking the conjectured form by using the following sum rule:

$$\sum_{m\geqslant 1} {}_N\langle\{m_i;n_j\}|p_m p_{-m}|\{m_i;n_j\}\rangle_N = p|\mu|. \tag{17}$$

The sum rule[3] follows from the so-called Sugawara form of the Virasoro operator L_0

$$L_0 = \frac{1}{2p}\rho_0^2 + \frac{1}{p}\sum_{m\geqslant 1}\rho_{-m}\rho_m. \tag{18}$$

The sum rule is given in terms of normalized basis states

$$|\{m_i;n_j\}\rangle_N = N_{\{m_i;n_j\}}^{-\frac{1}{2}}|\{m_i;n_j\}\rangle \tag{19}$$

with

$$N_{\{m_i;n_j\}} = \langle\{m_i;n_j\}|\{m_i;n_j\}\rangle = j_{\mu'}^{1/p} \tag{20}$$

with $\{\mu'\}$ the dual to the composite Young tableau defined in equation (15) and j the inner product given in equation (A.3).

We report in the following results for one- to three-particle states, in any Q-sector. In each case we give expressions for general p, and then specify to the case $p=2$, which is the case analysed numerically in section 4.

3.1. One quasi-particle

One finds

$$p_{-m}|m_1\rangle = \pm\sqrt{pg}|m_1 - m\rangle \tag{21}$$

which for $p=2$ gives

$$p_{-m}|\{m_1\}\rangle = -2|\{m_1 - m\}\rangle \qquad \text{for electrons} \tag{22}$$

$$p_{-m}|\{n_1\}\rangle = |\{n_1 - m\}\rangle \qquad \text{for quasi-holes.} \tag{23}$$

3.2. Two quasi-particles

At the level of two particles, many-body effects start to appear. It means that the density operator is not acting on each particle individually, but rather on both at the same time.

3.2.1. Two electrons or two quasi-holes. We obtain the following action:

$$p_{-m}|\{m_2,m_1\}\rangle = \pm m\sqrt{pg}\sum_{l=0}^{m} G(m_1,m_2;m_1-l,m_2-m+l)|\{m_2-m+l,m_1-l\}\rangle \tag{24}$$

$$\begin{aligned} G(m_1,m_2;m_1'&=m_1-l,m_2'=m_2-m+l) = G_{12} \\ &= \sum_{i=0}^{l}\frac{i(-)^i}{l(m-l+i)}\binom{m-l+i}{i}\binom{l}{i} \\ &\quad\times\frac{\Gamma(g+i)\Gamma(m_2-m_1+g+1)\Gamma(m_2'-m_1'+g-i)}{\Gamma(g-i)\Gamma(m_2-m_1+g+i+1)\Gamma(m_2'-m_1'+g)}. \end{aligned} \tag{25}$$

[3] Sum rule (17) can also be obtained using only Jack polynomial technology.

For expression equation (24) to make sense, we have to specify the meaning of two-particle states $|\{m_2, m_1\}\rangle$ with $m_2 < m_1$. For the case of electrons, the prescription is

$$\begin{aligned} |\{m_2', m_1'\}\rangle \quad &\Rightarrow \quad (-)^p \frac{N_{\{m_2', m_1'\}}}{N_{\{m_1'-p, m_2'+p\}}} |\{m_1' - p, m_2' + p\}\rangle \qquad \text{if} \quad m_2' \leqslant m_1' - 2p \\ &\Rightarrow \quad 0 \qquad \text{if} \quad m_2' - m_1' = -1, -2, \ldots, -2p+1. \end{aligned} \tag{26}$$

This rule can be extended to multi-particle states, and to quasi-holes. The latter behave slightly differently: one needs at least $(p+1)$ quasi-holes to have scattered states. Clearly, the prefactor $\frac{N_{\{m_2', m_1'\}}}{N_{\{m_1'-p, m_2'+p\}}}$ is analogous to the scattering phase that one expects in the continuum limit of this theory.

In the case of $p = 2$, we have

$$\begin{aligned} p_{-m}|\{m_2, m_1\}\rangle = -2 \Bigg(&|\{m_2 - m, m_1\}\rangle + \frac{(m_2 - m_1 + 1)(m_2 - m_1 + m + 3)}{(m_2 - m_1 + 3)(m_2 - m_1 + m + 1)} |\{m_2, m_1 - m\}\rangle \\ &- 2m \sum_{l=1}^{m-1} \frac{1}{(m_2 - m_1 + 3)(m_2 - m_1 + 2l - m + 1)} |\{m_2 - m + l, m_1 - l\}\rangle \Bigg) \end{aligned} \tag{27}$$

for electrons, and

$$\begin{aligned} p_{-m}|\{n_2, n_1\}\rangle = {}& |\{n_2 - m, n_1\}\rangle \\ &+ \frac{\Gamma(n_2 - n_1 + 1/2)\Gamma(n_2 - n_1 + 3/2)\Gamma^2(n_2 - n_1 + m + 1)}{\Gamma(n_2 - n_1 + m + 1/2)\Gamma(n_2 - n_1 + m + 3/2)\Gamma^2(n_2 - n_1 + 1)} \\ &\times |\{n_2, n_1 - m\}\rangle + \sum_{l=1}^{m-1} \left(\sum_{i=0}^{l} \frac{mi}{l(m - l + i)} (-)^i \binom{m - l + i}{i} \binom{l}{i} \frac{\Gamma^2(i + 1/2)}{\Gamma^2(1/2)} \right. \\ &\left. \times \frac{\Gamma(n_2 - n_1 + 3/2)\Gamma(n_2 - n_1 + 2l - m - i + 1/2)}{\Gamma(n_2 - n_1 + i + 3/2)\Gamma(n_2 - n_1 + 2l - m + 1/2)} \right) |\{n_2 - m + l, n_1 - l\}\rangle \end{aligned} \tag{28}$$

for quasi-holes. The factor appearing in front of the second term in the right-hand side of these expressions corresponds to the reordering of the particles during the action of the density operator. We shall call it a scattering factor, though it does not have the same physical origin as the one appearing in (26).

3.2.2. One electron and one quasi-hole.

$$\begin{aligned} p_{-m}|\{m_1; n_1\}\rangle = {}& -p|\{m_1 - m; n_1\}\rangle + \frac{(m_1 + p(n_1 - m) + 1)(m_1 + p(n_1 + 1))}{(m_1 + p(n_1 - m + 1))(m_1 + pn_1 + 1)} \\ &\times |\{m_1; n_1 - m\}\rangle - mp(p-1) \\ &\times \sum_{l=1}^{m-1} \frac{1}{(m_1 - m + l + p(n_1 - l + 1))(m_1 + 1 + pn_1)} |\{m_1 - m + l; n_1 - l\}\rangle \end{aligned} \tag{29}$$

along with the replacement

$$|\{-1; 0\}\rangle^Q \equiv \begin{cases} |\{-; -\}\rangle^{-(p-1)} & \text{for } Q = 0 \\ |\{0; -\}\rangle^{Q+1} & \text{for } Q < 0. \end{cases} \tag{30}$$

A G-function can also be defined here, defining it as the factor in the last line of (29) divided by m.

3.3. Three quasi-particles

For the case of form factors with three quasi-particles, we do not have complete results, but we report the following. We distinguish between terms where a single one, or two, or all three quasi-particles are affected by the action of the density operator

- when the density operator is affecting only one or two of the quasi-particles, one obtains the one- or two-particle action given above, multiplied by specific scattering factors
- when the density operator acts on all three quasi-particles, more complicated many-body effects appear for $p \neq 1$. One has to define a new function

$$G'_{ij} = \sum_{i=0}^{l-1} (-)^i \binom{m-l+i-1}{i} \binom{l-1}{i} \times \frac{\Gamma(g+i)\Gamma(m_2-m_1+g+1)\Gamma(m'_2-m'_1+g-i)}{\Gamma(g-i)\Gamma(m_2-m_1+g+i+1)\Gamma(m'_2-m'_1+g)} \tag{31}$$

for two particles of the same type. Here follow, case by case, the results we could obtain:

(2,1). Acting on $|\{m_2, m_1, m_0 = n_1\}\rangle$, the factor multiplying $|\{m'_2, m'_1, m'_0\}\rangle$ is

$$m[\delta m_0 G_{01} G_{02} G'_{12} + \delta m_1 G_{01} G_{12} + \delta m_2 G_{02} G_{12} + (\delta m_0 \delta m_1 + \delta m_0 \delta m_2 + \delta m_1 \delta m_2 / p - \delta m_0 \delta m_1 \delta m_2 / (p-1)) G_{01} G_{02} G_{12}] \tag{32}$$

with $\delta m_i = m_i - m'_i$ and the sum of the δm_i being equal to m. Replacements similar to (30) have to be made, which is the case for any mixed state.

(1,2). Acting on $|\{n_2, n_1, n_0 = m_1\}\rangle$, the factor multiplying $|\{n'_2, n'_1, n'_0\}\rangle$ is

$$m[\delta n_0 G_{01} G_{02} G'_{12} / p(1-p) + \delta n_1 G_{01} G_{12} + \delta n_2 G_{02} G_{12} + (\delta n_1 \delta n_2 - \delta n_0 \delta n_1 \delta n_2 / (p-1)) G_{01} G_{02} G_{12}]. \tag{33}$$

(3,0). Acting on $|\{m_3, m_2, m_1\}\rangle$, the factor multiplying $|\{m'_3, m'_2, m'_1\}\rangle$ is, for the special case $p = 2$,

$$-2m[\delta m_1 G_{12} G_{13} + \delta m_2 G_{12} G_{23} + \delta m_3 G_{13} G_{23} + (\delta m_1 \delta m_2 - \delta m_2 \delta m_3 + 3/2 \delta m_1 \delta m_2 \delta m_3) G_{12} G_{13} G_{23}]. \tag{34}$$

In the next section the various form factors presented here will be used in a form-factor expansion for the finite-temperature density–density correlation function.

4. Form-factor expansion

We now turn to the goal of computing a finite-temperature correlation function. Formally, it amounts to compute

$$\langle \mathcal{O} \rangle_T = \frac{1}{Z(T)} \sum_{\Psi \in \mathcal{H}_N} \langle \Psi | \mathcal{O} | \Psi \rangle \exp(-\beta E_\Psi) \tag{35}$$

where $\mathcal{H}_N$ is the Hilbert space of normalized states. The problem arising with this expression is that it is hard to handle for any given quantum field theory. Indeed, divergences appear in the correlators of the right-hand side and these need to be resummed. In the context of (massive) integrable field theory (IFT), it has been proposed that, using the basis of the asymptotic particle states in the zero-temperature theory, one may be able to rewrite (35) as a single sum free of divergences [7]. The resulting formula, called a 'form-factor expansion', can be evaluated by using scattering data, the thermodynamic Bethe ansatz (TBA) and the form-factor bootstrap (FFB) (see [18] for a discussion of how the TBA is recovered from the FFB). There is an ongoing discussion about how precisely the form-factor expansion can be implemented in IFTs, in particular for the case of multi-point correlation functions [7–12].

For the case of conformal field theory, there has been a similar but independent proposal for writing finite-temperature correlators in terms of quasi-particle form factors and appropriate thermal distribution functions [6]. The idea here is that the 'fqH basis' (13) of eigenstates of the CS Hamiltonian provides the proper notion of 'asymptotic particle states', with simple, but non-trivial fractional statistics properties.

The motivation for studying the form-factor expansion for CFT has been that, if successful, this approach to finite-temperature correlation functions can possibly be extended to models, such as quantum spin chains, that are gapless but that are not CFTs.

Comparing the proposed form-factor expansions for IFT and CFT (the particular CFT discussed in this paper), one is led to identify a two-body S-matrix of the simple form

$$\boldsymbol{S} = \exp[2\mathrm{i}\pi(\boldsymbol{\delta} - \boldsymbol{g})\Theta(\theta)] \tag{36}$$

in the thermodynamic limit. It is still an open question whether the form factor computed in [6] and in this paper can be obtained by means of an axiomatic approach, starting from these scattering data.

In this paper, we study the following CFT form-factor expansion for one-point functions

$$\langle \mathcal{O}(\varepsilon)\rangle_T = \frac{1}{\beta}\sum_{M,N} O^{(M,N)}(\varepsilon) \tag{37}$$

$$O^{(M,N)}(\varepsilon) = \beta a \sum_{\{m_i;n_j\}} D^{(M,N)}(m,\{m_i;n_j\}) \prod_{i=1}^{M} \bar{n}_p(am_i) \prod_{j=1}^{N} \bar{n}_{1/p}(an_j) \tag{38}$$

$$D^{(M,N)}(m,\{m_i;n_j\}) = {}_N\langle\{m_i;n_j\}|\mathcal{O}(m)|\{m_i;n_j\}\rangle_N + \text{subfactors}. \tag{39}$$

Here $a = 2\pi/L$, $\varepsilon = am$, $\mathcal{O}(\varepsilon) = a\mathcal{O}(m)$; the continuum limit is obtained by sending $a \to 0$. D is the irreducible form factor, which we shall define with precision. In (39), the subfactors are built from multi-particle states which are subsets of $\{m_i; n_j\}$. Their leading state of the form (4) is the original one on which some creation operators are cancelled. It has then to be rewritten with the correct charge sector. Specific examples of our definitions are,

$$\begin{aligned} D^{(2,0)}(m,\{m_2,m_1\}^Q) \equiv {}^Q_N\langle\{m_2,m_1\}|\mathcal{O}(m)|\{m_2,m_1\}\rangle^Q_N - {}^Q_N\langle\{m_1\}|\mathcal{O}(m)|\{m_1\}\rangle^Q_N \\ - {}^Q_N\langle\{m_2+p\}|\mathcal{O}(m)|\{m_2+p\}\rangle^Q_N, \end{aligned} \tag{40}$$

$$\begin{aligned} D^{(0,2)}(m,\{n_2,n_1\}^Q) \equiv {}^Q_N\langle\{n_2,n_1\}|\mathcal{O}(m)|\{n_2,n_1\}\rangle^Q_N - {}^Q_N\langle\{n_1\}|\mathcal{O}(m)|\{n_1\}\rangle^Q_N \\ - {}^{Q+1(-p)}_N\langle\{n_2(+1)\}|\mathcal{O}(m)|\{n_2(+1)\}\rangle^{Q+1(-p)}_N, \end{aligned} \tag{41}$$

the charge shift $(-p)$ and the momentum shift (+1) being present for $Q = 0$.

This definition can be interpreted as follows. Each form factor may appear as part of bigger form factors. The way to resum them is to use these irreducible form factors and the one-particle distributions. This expansion has already proved successful in describing the Green function for the $\nu = 1/p$ fqH [6]. In the following we will concentrate on the density–density correlation function, where $\mathcal{O}(m) = p_m p_{-m}$. Most of the results will be given for any p, we will restrict to the $p = 2$ case for numerics.

Using standard CFT methods, one finds the density–density correlator to be

$$\langle\rho(-\varepsilon)\rho(\varepsilon)\rangle_T = \frac{1}{\beta}\frac{p\beta\varepsilon}{\mathrm{e}^{\beta\varepsilon}-1}. \tag{42}$$

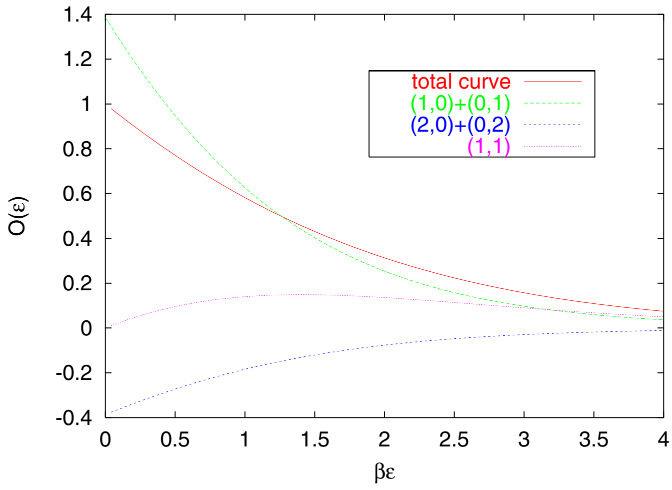

Figure 1. All the form-factor contributions at $p = 1$.

We shall compare the results of the form-factor expansion with this exact result. We start (section 4.1) by explaining how the procedure works for $p = 1$. It already contains all the features and is completely solvable. Then (sections 4.2 and 4.3), we treat the general p case, where we evaluate the asymptotics of the form-factor expansion for $\beta\varepsilon \to 0$ or ∞ in closed form. Finally (section 4.4), we discuss numerical results for the full correlation function for the case $p = 2$.

4.1. Case p = 1

The irreducible form factors are here straightforward to obtain. One finds the following contributions to the density–density correlator

$$O^{(1,0)} = O^{(0,1)} = \beta \int_{\varepsilon}^{\infty} \mathrm{d}\varepsilon_1 \bar{n}_1(\varepsilon_1) = \log\left(\frac{\mathrm{e}^{\beta\varepsilon}+1}{\mathrm{e}^{\beta\varepsilon}}\right) \tag{43}$$

$$O^{(2,0)} = O^{(0,2)} = -\beta \int_{0}^{\infty} \mathrm{d}\varepsilon_1 \bar{n}_1(\varepsilon_1)\bar{n}_1(\varepsilon+\varepsilon_1) = \frac{1}{\mathrm{e}^{\beta\varepsilon}-1}\left[\log 2 - \mathrm{e}^{\beta\varepsilon}\log\left(\frac{\mathrm{e}^{\beta\varepsilon}+1}{\mathrm{e}^{\beta\varepsilon}}\right)\right] \tag{44}$$

$$O^{(1,1)} = \beta \int_{0}^{\varepsilon} \mathrm{d}\varepsilon_1 \bar{n}_1(\varepsilon_1)\bar{n}_1(\varepsilon-\varepsilon_1) = \frac{2}{\mathrm{e}^{\beta\varepsilon}-1}\log\left(\frac{\mathrm{e}^{\beta\varepsilon}+1}{2\,\mathrm{e}^{\beta\varepsilon/2}}\right). \tag{45}$$

All the other irreducible form factors vanish. One then easily checks that the sum of all contributions gives the correct answer $\beta\varepsilon/(\exp(\beta\varepsilon)-1)$. The corresponding curves are shown in figure 1.

Before going to general p, let us remark a few facts:

- the asymptotics for $\beta\varepsilon \to 0$ are
  - $\beta \int_0^\infty \mathrm{d}\varepsilon\, \bar{n}_1(\varepsilon)$ for one electron and one quasi-hole
  - $-\beta \int_0^\infty \mathrm{d}\varepsilon\, \bar{n}_1^2(\varepsilon)$ for two electrons or two quasi-holes
  - 0 for one electron and one quasi-hole;

  giving a total of $2\int_0^\infty \mathrm{d}\beta\varepsilon\, \bar{n}_1(\varepsilon)(1-\bar{n}_1(\varepsilon)) = 2\bar{n}_1(0) = 1$.

- the asymptotics for $\beta\varepsilon \to \infty$ are
  - $\beta\,\mathrm{e}^{-\beta\varepsilon}$ for one electron or one quasi-hole
  - $\beta(\log 2 - 1)\,\mathrm{e}^{-\beta\varepsilon}$ for two electrons or two quasi-holes
  - $\beta\varepsilon\,\mathrm{e}^{-\beta\varepsilon}$ for one electron and one quasi-hole: it is the dominant one, and fits the exact asymptote.

These features for the asymptotes will be shared for general p, as we will see now.

4.2. Low-temperature expansion

We look here at the $\varepsilon \to \infty$ limit, where the correlation function behaves as $p\beta\varepsilon\,\mathrm{e}^{-\beta\varepsilon}$.

For a generic multi-particle excitation, this limit will be dominated by excitations for which one quasi-particle has a momentum m_i bigger than $m = \varepsilon/a$, and all the others a much smaller one. The main contribution comes from the term where the density operator acts only on the quasi-particle of momentum m_i. This contribution, which appears equally in all the subfactors of the irreducible form factor, is proportional to $\varepsilon^{1-g}\exp(-\beta\varepsilon)$. The contribution from each other particle is of the form

$$\text{contribution }(\varepsilon_{j\neq i}) \propto \int_0^\infty \mathrm{d}\varepsilon_j \bar{n}_{g_j}(\varepsilon_j)(S_{ij} - 1) \tag{46}$$

$$\propto \int_0^\infty \mathrm{d}\varepsilon_j \bar{n}_{g_j}(\varepsilon_j)\frac{1}{(\varepsilon_i - \varepsilon)/\sqrt{g_i} \pm \varepsilon_j/\sqrt{g_j}} \tag{47}$$

where S_{ij} is a scattering factor. These parts do not affect the leading behaviour for $\varepsilon \to \infty$ and one finds a total contribution which falls off faster than $\varepsilon\,\mathrm{e}^{-\beta\varepsilon}$.

There is one exception to the above-mentioned rule. The form factor can become big if the intermediate state is the vacuum, that is for an initial neutral excitation consisting of one electron and p quasi-holes. To see this we use the expansion of p_m in terms of Jack polynomials: we remark that for $\lambda = 1/p$ this expansion is indeed restricted to states with one electron and p quasi-holes. The factor is then quickly found to be

$$D(1,2)\left(m = m_1 + \sum_{j=1}^{p} n_j + p; m_1; \{n_j\}\right) = \left(\chi_\nu^{1/p}\right)^2 j_\nu^{1/p} \tag{48}$$

with $\{\nu\} = (\{n_j + 1\}, 1^{m_1})$. This expression has been evaluated in [19]. In the thermodynamic limit $a \to 0$ it gives

$$O^{(1,p)}(\varepsilon) = \beta p^2 \frac{\Gamma(p)\Gamma^p(1/p)}{\prod_{i=1}^p \Gamma^2(i/p)} \int \mathrm{d}\varepsilon_1 \prod_{i=1}^{p} \mathrm{d}\varepsilon_i'\delta\left(\varepsilon - \varepsilon_1 - \sum_i \varepsilon_i'\right) \frac{\varepsilon^2\varepsilon_1^{p-1}\prod_{i<j}(\varepsilon_j' - \varepsilon_i')^{2/p}}{\prod_i(\varepsilon_1 + p\varepsilon_i')^2\prod_i(\varepsilon_i')^{1-1/p}}$$
$$\times\, \bar{n}_p(\varepsilon_1)\prod_i \bar{n}_{1/p}(\varepsilon_p') + \text{subdominant terms.} \tag{49}$$

Power counting in this formula gives an asymptotic behaviour proportional to $\varepsilon\,\mathrm{e}^{-\beta\varepsilon}$. The coefficient in front is obtained by remarking that

$$O_T^{(1,p)}(\varepsilon) \simeq O_0^{(1,p)}(\varepsilon)\,\mathrm{e}^{-\beta\varepsilon}. \tag{50}$$

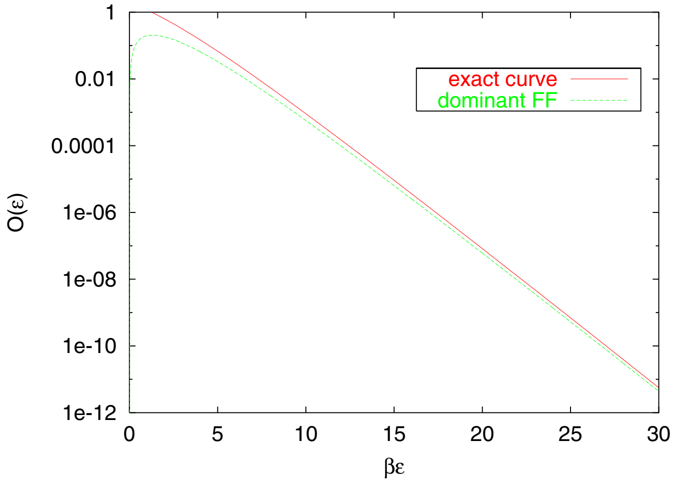

Figure 2. Comparison of the exact $p = 2$ low-temperature asymptote with the contribution from the form factor for a (1,2) state.

For example, for $p = 2$

$$O^{(1,2)}(\varepsilon) \simeq \beta\varepsilon\,\mathrm{e}^{-\beta\varepsilon}4\int \mathrm{d}\varepsilon_1\,\mathrm{d}\varepsilon_1'\,\mathrm{d}\varepsilon_2'\frac{\varepsilon\varepsilon_1(\varepsilon_2'-\varepsilon_1')}{\sqrt{\varepsilon_1'\varepsilon_2'}(\varepsilon_1+2\varepsilon_1')^2(\varepsilon_1+2\varepsilon_2')^2}$$

$$\times\,\Theta(\varepsilon_2'-\varepsilon_1')\delta(\varepsilon-\varepsilon_1-\varepsilon_1'-\varepsilon_2') \tag{51}$$

$$\simeq \beta\varepsilon\,\mathrm{e}^{-\beta\varepsilon}4\int_0^\infty \mathrm{d}\theta\frac{\cosh(\theta)}{\sinh^3(\theta)}[\sinh(\theta)-\theta] \tag{52}$$

$$= 2\beta\varepsilon\,\mathrm{e}^{-\beta\varepsilon} \tag{53}$$

which is the right answer. The result is shown in figure 2.

4.3. High-temperature expansion

When contemplating a form-factor expansion for finite-temperature correlators, one may worry about the convergence in the high-temperature ($\beta\varepsilon \to 0$) limit, where thermal distribution functions do not effectively suppress many-particle contributions. We will see that in our case the situation is remarkably good: using form factors with up to three quasi-particles, we recover the exact high-temperature limit. To establish this result, we use the following identity for the equilibrium distribution function $\bar{n}_g$ [16]

$$\bar{n}_g(\varepsilon) + (1-2g)\bar{n}_g^2(\varepsilon) - g(1-g)\bar{n}_g^3(\varepsilon) = -\bar{n}_g'(\varepsilon). \tag{54}$$

4.3.1. One particle. Direct application of (21) gives the irreducible form factor

$$D^{(1)}(m; m_1) = pg\frac{\Gamma(m_1-m+g)\Gamma(m_1+1)}{\Gamma(m_1-m+1)\Gamma(m_1+g)}. \tag{55}$$

The form-factor contribution is then

$$O^{(1)}(\varepsilon\to 0) = pg\beta\int \mathrm{d}\varepsilon_1\bar{n}_g(\varepsilon_1). \tag{56}$$

The curves obtained at $p = 2$ for one electron and one quasi-hole are given in figure 3.

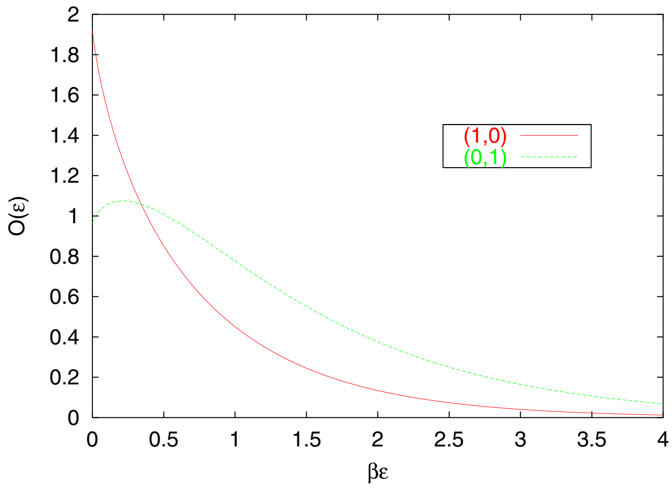

Figure 3. One-particle contributions at $p = 2$.

4.3.2. Two particles. Using (24) and the definitions for the irreducible form factor (40), (41), one obtains for $(m_1, m_2) \gg m$ (two particles of the same type)

$$D^{(2)}(m; m_2, m_1) = pg2 \sum_{i=1}^{m} (-)^i \binom{m-1}{i-1} \binom{m+i-1}{i} \times \frac{\Gamma(g+i)\Gamma(m_2 - m_1 + g - i)\Gamma(m_2 - m_1 + g + 1)}{\Gamma(g-i)\Gamma(m_2 - m_1 + g)\Gamma(m_2 - m_1 + g + i)} \tag{57}$$

$$\simeq pg\delta_{m_2,m_1}(2g-1) \sum_{i=1}^{m} \frac{(-)^i}{2i-1} \binom{m-1}{i-1} \binom{m+i-1}{i} \tag{58}$$

$$\simeq -pg(2g-1)\delta_{m_2,m_1} \tag{59}$$

leading to

$$O^{(2)}(\varepsilon \to 0) = -pg(2g-1)\beta \int \mathrm{d}\varepsilon_1 \bar{n}_g^2(\varepsilon_1). \tag{60}$$

The curves for two electrons and two quasi-holes at $p = 2$ are given in figure 4. As can be seen, the limit $\varepsilon \to 0$ is indeed $-12\beta \int \mathrm{d}\varepsilon_1\, \bar{n}_2^2(\varepsilon_1) = -0.9472$ for two electrons, and 0 for two quasi-holes.

For mixed states (one electron and one quasi-hole), (29) leads to the following limit for the irreducible form factor:

$$O^{(1,1)}(\varepsilon \to 0) \sim (p^2+1)\beta\varepsilon \int_{\varepsilon} \mathrm{d}\varepsilon_1\, \mathrm{d}\varepsilon_1' \frac{1}{(\varepsilon_1 + p\varepsilon_1')^2} \bar{n}_p(\varepsilon_1)\bar{n}_{1/p}(\varepsilon_1') \tag{61}$$

leading to a linear in ε dependence. So it has a vanishing limit for $\varepsilon \to 0$. This result will be the same for any mixed state.

4.3.3. Three particles. Considering the argument given above, we are only interested in the contribution of three particles of the same type (three electrons or three quasi-holes). As for two particles, we find that the contribution in the limit $\beta\varepsilon \to 0$ comes entirely from the

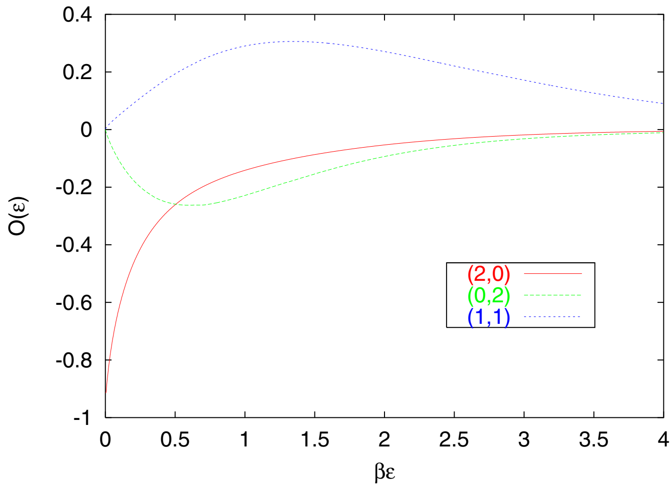

Figure 4. Two-particle contributions at $p = 2$.

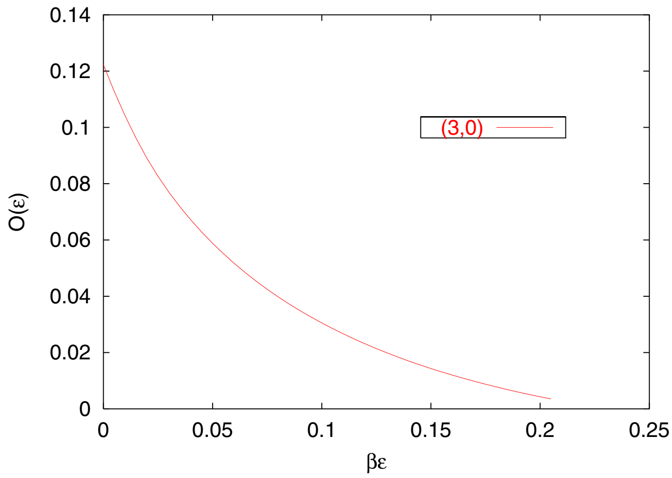

Figure 5. Three-particle contribution at $p = 2$ in the high-temperature limit. Due to the complexity of the numerical calculations, the discretization used for $\beta\varepsilon$ is 5×10^{-3}, a relatively large value which accounts for the underestimation of the $\varepsilon \to 0$ limit.

diagonal, where the energies of the three particles are the same. We have not been able to get an analytic expression for the high-temperature limit, but we present the following conjecture:

$$O^{(3)}(\varepsilon \to 0) = pg^2(g-1)\beta \int \mathrm{d}\varepsilon_1 \bar{n}_g^3(\varepsilon_1). \tag{62}$$

We have strong support for this conjecture. First of all, thanks to (34), we are able to compute the contributions from form factors for three electrons at $p = 2$ for low values of $m > 0$, all representing the limit $\varepsilon = am \to 0$. We reproduce the expected form (62) when $m = 3$, and we expect that this result is stable for $m \geqslant 3$. Continuity for bigger values of m (corresponding to finite $\varepsilon = am$) has been checked numerically, as is shown in figure 5. (We note that according to equation (62) the numerical value of the (3, 0) contribution at $\varepsilon = 0$ will be 0.1272; the numerical curve displayed in figure 5 gives a slightly smaller value, due

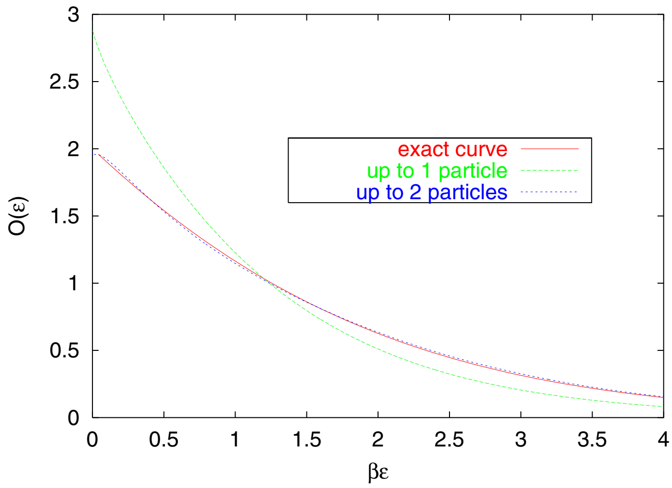

Figure 6. High-temperature expansion. One can compare the exact density–density correlation function at $p = 2$ and the contributions from up to one and two quasi-particles. The agreement is very good in the latter case.

to the fact that a relatively large value of the discretization a had to be used.) Further support for the conjecture comes from the irreducible form factor for three quasi-holes for general p at $m = 1$, which directly gives expression (62).

4.3.4. Sum of one, two and three-particle contributions. We are now ready to evaluate the sum of all contributions to the density–density correlator at $\beta\varepsilon \to 0$. Adding the contributions from form factors with one, two, or three quasi-particles of statistics g results in

$$pg\beta \int_0^\infty \mathrm{d}\varepsilon \left(\bar{n}_g(\varepsilon) + (1-2g)\bar{n}_g^2(\varepsilon) - g(1-g)\bar{n}_g^3(\varepsilon)\right) = -pg\beta \int_0^\infty \mathrm{d}\varepsilon \bar{n}_g'(\varepsilon) = pg\bar{n}_g(0). \tag{63}$$

The density–density correlator has these contributions from electrons ($g = p$) and from quasi-holes ($g = 1/p$). Through duality $g\,\bar{n}_g(0) + 1/g\,\bar{n}_{1/g}(0) = 1$, leading to the result

$$\beta\langle\rho(-\varepsilon)\rho(\varepsilon)\rangle_T \overset{\beta\varepsilon\to 0}{\to} p \tag{64}$$

in agreement with the exact result. We thus find that the high-temperature limit is saturated by contributions with up to three quasi-particles, with the three-particle contributions being absent in the special case $p = 1$. In our view, this non-trivial, exact result gives strong support for the validity of the proposed CFT form-factor expansion. We mention that, in general, good convergence at the high-temperature end is scarcely seen in form-factor expansions.

4.4. Full correlation function

In the two preceding sections, we have proved that the low- and high-temperature asymptotics for the density–density correlation function were recovered in the form-factor expansion using only a few quasi-particles. We now observe that the integrated value of the correlator is recovered already at the level of the one particle contributions. One only needs sum rule (17) and the procedure for building irreducible form factors to prove this. At the level of one particle, one gets

$$\beta \int_0^\infty \mathrm{d}\varepsilon\, O^{(1)} = p\beta \int_0^\infty \mathrm{d}\varepsilon\, \beta\varepsilon(\bar{n}_p + \bar{n}_{1/p})(\varepsilon) = p\beta \int_0^\infty \mathrm{d}\varepsilon \frac{\beta\varepsilon}{\mathrm{e}^{\beta\varepsilon} - 1}. \qquad (65)$$

At the level of two particles,

$$\beta \int_0^\infty \mathrm{d}\varepsilon\, O^{(2)} = -\frac{1}{2}\left[\beta \int_0^\infty \mathrm{d}\varepsilon(p\bar{n}_p - \bar{n}_{1/p})(\varepsilon)\right]^2 = 0 \qquad (66)$$

by duality. For any contribution with more than two particles, the integrated curve is 0.

All these facts tend to show that the proposed form-factor expansion gives a quick convergence for the *full* correlation function. This is indeed the case, as one sees on figure 6, where the contribution from states with up to two particles at $p = 2$ have been collected.

5. Conclusion

We have developed a form-factor expansion for the density–density correlation function in the fqH edge CFT at $\nu = 1/p$. We have used the correspondence between a fqH basis made of dual quasi-particles (electrons and quasi-holes) and the Jack polynomial basis of the Calogero–Sutherland model. We used 'Jack polynomial technology' to study form factors, and obtained analytical expressions for form factors with up to three excited particles. Using these form factors in the proposed form-factor expansion, we showed that the convergence towards the exact correlation function is quick in terms of the number of excited particles. We demonstrated that in the high- and low-temperature regimes, the expansion reproduces the exact result. This is a clear indication that the expansion is correct.

This result opens the way for the computation of correlation functions for non-CFT theories, as soon as their excitations form a gas of fractional statistics particles. This is the case for the Calogero–Sutherland model, and the models derived from it, like the Haldane–Shastry spin chain. Of fundamental interest also is the need to understand the connection between the thermodynamic limit of the CFT form factors and a thermodynamic Bethe ansatz method. This would allow for treating such complicated problems as impurities in edge-to-edge tunnelling.

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Appendix. Jack polynomial technology

Most of the Jack polynomial technology is found in [20]. We reproduce here the results useful in the present paper. Jack polynomials form a complete and orthogonal basis of the ring of symmetric polynomials with the given inner product:

$$\langle p_\mu, p_\nu \rangle = \delta_{\mu,\nu} \lambda^{-l(\mu)} z_\mu \qquad (A.1)$$

where $p_\mu = \prod_{j=1}^{l(\mu)} p_{\mu_j}$, $p_m(\{x_i\}) = \sum_i x_i^m$ are the power sums, and $z_\mu = \prod_{i\geqslant 1} i^{m_i} m_i!$

They are defined in a unique way through their development in monomial symmetric functions

$$P^{\lambda}_{\mu} = \sum_{\nu \leqslant \mu} v^{\lambda}_{\mu,\nu} m_{\nu} \qquad \text{with} \quad v_{\mu,\mu} = 1. \tag{A.2}$$

Their norms are

$$\left\langle P^{\lambda}_{\mu}, P^{\lambda}_{\mu} \right\rangle = j^{\lambda}_{\{\nu\}} = \prod_{s \in \{\nu\}} \frac{\lambda l(s) + a(s) + 1}{\lambda(l(s)+1) + a(s)} \tag{A.3}$$

where $l(s)$ and $a(s)$ are the leg and the arm of cell s in the tableau $\{\nu\}$.

In the paper, we need the expansion of power sums in Jack polynomials

$$p_m = \sum_{\mu \vdash m} \chi^{\lambda}_{\mu} J^{\lambda}_{\mu} \tag{A.4}$$

$$\chi^{\lambda}_{\mu} = m \frac{\prod_{s \neq (1,1)} (a'(s) - \lambda l'(s))}{\prod_{s} (\lambda l(s) + a(s) + 1)} \tag{A.5}$$

and in elementary polynomials:

$$p_m = m \sum_{\nu \vdash m} (-1)^{m - l(\nu)} \frac{(l(\nu) - 1)!}{\prod_i m_i(\nu)!} e_{\nu} \tag{A.6}$$

where $e_{\nu} = e_{\nu_1} \ldots e_{\nu_m}$ and $e_r = J^{\lambda}_{(1^r)}$ for any λ. For example,

$$p_1 = e_1 \tag{A.7}$$

$$p_2 = e_{1^2} - 2e_2 \tag{A.8}$$

$$p_3 = e_{1^3} - 3e_{21} + 3e_3 \tag{A.9}$$

$$p_4 = e_{1^4} - 4e_{21^2} + 2e_{2^2} + 4e_{31} - 4e_4 \tag{A.10}$$

$$p_5 = e_{1^5} - 5e_{21^3} + 5e_{2^21} + 5e_{31^2} - 5e_{32} - 5e_{41} + 5e_5 \tag{A.11}$$

$$p_6 = e_{1^6} - 6e_{21^4} + 9e_{2^21^2} - 2e_{2^3} + 6e_{31^3} - 12e_{321} + 3e_{3^2} - 6e_{41^2} + 6e_{42} + 6e_{51} - 6e_6. \tag{A.12}$$

Inner products between Jacks and elementary symmetric functions are known through the Pieri formula

$$J^{\lambda}_{\nu} e_r = \sum_{\mu} \psi'_{\mu/\nu} J^{\lambda}_{\mu} \tag{A.13}$$

$$\psi'_{\mu/\nu} = \prod_{C_{\mu/\nu} \setminus R_{\mu/\nu}} \frac{j^{\lambda}_{\mu}(s)}{j^{\lambda}_{\nu}(s)} \tag{A.14}$$

with $\mu - \nu$ being a vertical r-strip (at maximum one box per row, for a total of r), $C_{\mu/\nu}$ (resp $R_{\mu/\nu}$) being the union of columns (resp rows) that intersect $\mu - \nu$.

References

[1] Haldane F 1991 *Phys. Rev. Lett.* **67** 937
[2] van Elburg R and Schoutens K 1998 *Phys. Rev.* B **58** 15704
[3] Berkovich A and McCoy B 1999 Statistical physics on the eve of the 21st century *Advances in Statistical Mechanics* ed M Batchelor and L Wille vol 14 (Singapore: World Scientific) pp 240–56
[4] Schoutens K 1997 *Phys. Rev. Lett.* **79** 2608
[5] Iso S 1995 *Nucl. Phys.* B **443** 581

[6] van Elburg R and Schoutens K 2000 *J. Phys. A: Math. Gen.* **33** 7987
[7] LeClair A and Mussardo G 1999 *Nucl. Phys.* B **552** 624
[8] Delfino G 2001 *J. Phys. A: Math. Gen.* **34** L161
[9] Mussardo G 2001 *J. Phys. A: Math. Gen.* **34** 7399
[10] Konik R 2001 Haldane gapped spin chains: exact low temperature expansions of correlation functions *Preprint* cond-mat/0105284
[11] Saleur H 2000 *Nucl. Phys.* B **567** 602
[12] Castro-Alvaredo O A and Fring A 2002 Finite temperature correlation functions from form factors *Preprint* hep-th/0203130
[13] Dasnières de Veigy A and Ouvry S 1994 *Phys. Rev. Lett.* **72** 600
[14] Isakov S 1994 *Mod. Phys. Lett.* B **8** 319
[15] Wu Y 1994 *Phys. Rev. Lett.* **73** 922
[16] Isakov S, Martin T and Ouvry S 1999 *Phys. Rev. Lett.* **83** 580
[17] Lesage F, Pasquier V and Serban D 1995 *Nucl. Phys.* B **435** 585
[18] Balog J 1994 *Nucl. Phys.* B **419** 480
[19] Ha Z 1995 *Nucl. Phys.* B **435** 604
[20] Macdonald I 1995 *Symmetric Functions and Hall Polynomials* (Clarendon: Oxford)